Find the minimum value of a positive real number, y such that:

y = √ ((x+6)2 + 25) + √ ((x-6)2 + 121)

To find the minimum value of y (a function of x), find the value of x as dy/dx 🡺 0 and substituting the value(s) of x in the function y.

Let u = (x+6)2 + 25

And v = (x-6)2 + 121

Therefore, y = √(U) +√(V).

From the differential laws; differentiating a function of a function:

If U is a function of x, i.e. U(x)

And y = U2

dy/dx = dy/dU × dU/dx

let u = (x+6)2 + 25

and v = (x-6)2 + 121

y = √((x+6)2 + 25) + √((x-6)2 + 121) \*\*\*\*\*\*\*\*eq.(1)

y = √(u) +√(v) \*\*\*\*\*\*\*\*eq.(2)

dy/du = 1/(2×√(U))

dy/dv = 1/(2×√(V))

du/dx = 2x + 12

dv/dx = 2x – 12

dy/dx = (dy/du × du/dx)+ (dy/dv × dv/dx)

Substituting the values of the differentials in \*\*\*\*\*\*\*\*eq.(2):

dy/dx =(2x+12)/(2×√((x+6)2+121)) + (2x-12)/(2×√((x-6)2+25))

dy/dx =(x+6)/(√((x+6)2+121)) + (x-6)/(√((x-6)2+25))

Recall as dy/dx 🡺 0

0=(x+6)/(√ ((x+6)2+121)) + (x-6)/(√ ((x-6)2+25))

-(x-6)/(√((x-6)2+25)) = (x+6)/(√((x+6)2+121)) \*\*\*\*\*\*\*\*eq.(3)

taking the square of both sides of \*\*\*\*\*\*\*\*eq.(3)

(x-6)(x-6)/((x-6)2+25) = (x+6)×(x+6)/((x+6)2+121) \*\*\*\*\*\*\*\*eq.(4)

Multiplying through by the product of the denominators:

(x-6)2×[(x+6)2+121] = (x+6)2 ×[(x-6)2+25]

(x2-36)2+[121× (x+6)2] = (x2-36)2 ×[25×(x-6)2] \*\*\*\*\*\*\*\*eq.(5)

Subtracting (x2-36)2 from both sides of \*\*\*\*\*\*\*\*eq.(5)

121×(x+6)2 =25×(x-6)2

121(x2+12x+36) = 25(x2-12x+36)

121x2+1452x+4356 = 25x2-300x+900 \*\*\*\*\*\*\*\*eq.(6)

Subtracting the terms: 25x2, -300x, and 900 from both sides of the equality sign

96x2+1752x+3456 = 0 \*\*\*\*\*\*\*\*eq.(7)

Using the quadratic formula to resolve \*\*\*\*\*\*\*\*eq.(7)

x = [-b ± √((b×b)-4ac)]/2a

x = [-1752 **±** √{(1752×1752)-(4×96×3456)}]/(2×96)

As dy/dx 🡺 0,

x = -2.16 or -16

Substituting the values of x as dy/dx 🡺 0 in \*\*\*\*\*\*\*\*eq.(1)

x = -16

y = √((-16+6)2+25) + √((-16-6)2+121)

y = √125 + √605

y = 5√5 + 11√5

y = 16√5

x = -2.16

y = √((-2.16+6)2+25) + √((-2.16-6)2+121)

y = 19.9999

y ≈ 20.

**Therefore, the minimum value of y is 20!**